Important Theorems and Definitions concerning Derivatives

The Definition of the Derivative of a function at a point $(c, f(c))$	$f'(c) = \lim_{\Delta x \to 0}\left(\frac{f(c+\Delta x) - f(c)}{\Delta x}\right)$ The Alternative definition is $f'(c) = \lim_{x \to c}\left(\frac{f(x) - f(c)}{x - c}\right)$
Definition of the Derivative of a Function	$f'(x) = \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right)$ The Alternative definition is $f'(x) = \lim_{z \to x}\left(\frac{f(z) - f(x)}{z - x}\right)$
Notations for derivatives and higher order derivatives	$f'(x)$ $\quad \frac{d}{dx} f(x)$ $\quad D_x f(x)$ $\quad y'$ $\quad \frac{dy}{dx}$ $\quad D_x y$ $f''(x)$ $\quad \frac{d^2}{dx^2} f(x)$ $\quad D_{x^2} f(x)$ $\quad y''$ $\quad \frac{d^2 y}{dx^2}$ $\quad D_{x^2} y$
Differentiability Implies Continuity	If a function is differentiable at x = c then the function is continuous at x = c.
Causes of Non-differentiability	1. Discontinuity 2. Sharp turn or cusp 3. Vertical tangent
Basic Differential Rules	1. Constant Rule: $D_x[c] = 0$ where c is a constant 2. Power Rule: $D_x\left(x^n\right) = nx^{n-1}$ 3. Scalar Rule: $D_x\left(kf(x)\right) = kf'(x)$ 4. Sum or difference law $D_x[f(x) \pm g(x)] = f'(x) \pm g'(x)$ 5. Product Law $D_x\left(f(x)g(x)\right) = f'(x)g(x) + g'(x)f(x)$ 6. Quotient Law $D_x\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - g'(x)f(x)}{\left(g(x)\right)^2}$

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The Derivatives of Trig functions	1. $D_x(\sin x) = \cos x$ 2. $D_x(\cos x) = -\sin x$ 3. $D_x(\tan x) = \sec^2 x$ 4. $D_x(\cot x) = -\csc^2 x$ 5. $D_x(\sec x) = \sec x \tan x$ 6. $D_x(\csc x) = -\csc x \cot x$
The Chain Rule	If $y = f(u)$ is a differential function of u and $u = g(x)$ is a differential function of x, then $y = f(g(x))$ is a differential function of x and $\frac{dy}{dx} = \frac{dy}{du} \bullet \frac{du}{dx}$ (Leibniz notation) or $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$ (Newton's notation)
Derivative Rules of logarithms and exponential functions	$\frac{d}{dx}\ln\lvert x\rvert = \frac{1}{x}$ $\frac{d}{dx}\log_a x = \frac{1}{x\ln a}$ $\frac{d}{dx}e^x = e^x$ $\frac{d}{dx}a^x = a^x \ln a$
Derivatives of Inverse Trig Functions	1. $\frac{d}{dx}Arc\sin x = \frac{1}{\sqrt{1-x^2}}$ 2. $\frac{d}{dx}Arc\cos x = \frac{-1}{\sqrt{1-x^2}}$ 3. $\frac{d}{dx}Arc\tan x = \frac{1}{1+x^2}$ 4. $\frac{d}{dx}Arc\cot x = \frac{-1}{1+x^2}$ 5. $\frac{d}{dx}Arc\sec x = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$ 6. $\frac{d}{dx}Arc\csc x = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$
The Derivative of an Inverse Function	If $g(x)$ is the inverse function of $f(x)$ and $g(b) = a$ then $g'(b) = \frac{1}{f'(a)}$

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