Midpoint

$\pi/4\left(\sin\frac{\pi}{8} + \sin\frac{3\pi}{8} + \sin\frac{5\pi}{8} + \sin\frac{7\pi}{8}\right)$

$= 2.05$

Vocab:

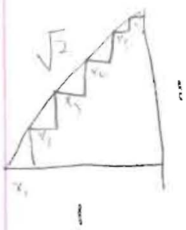
inscribe · inside boxes

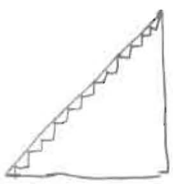
· Underaproximation

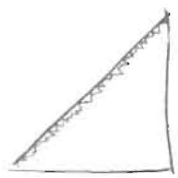
or

Circumscribe

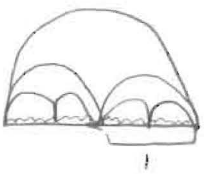
· over approximation







$y_1 + y_2 + y_3 + y_4 + y_5 = 1$ $\quad x_1 + x_2 + x_3 + \ldots x_{\text{tiny}} = 1$ $\quad x_1 + x_2 + x_3 + \ldots x_i + \ldots x_N = \sqrt{2}$



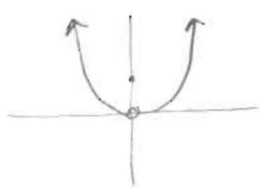
Arclength - π

π - 2

$f(x) = \begin{cases} x^2 & \text{if } x \neq 0 \\ 3 & \text{if } x = 0 \end{cases}$

$f'(x) = \begin{cases} 2x & \text{if } x \neq 0 \\ \text{und.} & \text{if } x = 0 \end{cases}$

$f'x$ - - - 0 + + + +

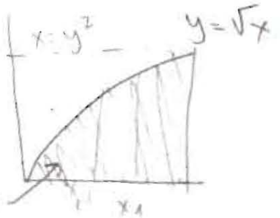


should say increasing $[0, \infty)$ b/c 0 is a critical point, even though not differentiable

$\sum_{K=3}^{100}$

list math sum seq $(Y_1, x, 3, 6)$

- Use Riemann sum on test
- Geometric interpretation circle, rect, triangle, square



$X_0 = 0$

$X_i = 0 + i\Delta X = \frac{i}{n}$

$\Delta X = \frac{1}{n}$

2/3

$\lim_{n\to\infty} \sum_{i=1}^{N} F(x_i)\Delta X$

$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\frac{1}{N}} \left(\frac{1}{N}\right)$

square is 1x1

$1 - \lim_{n\to\infty} \sum_{i=y}^{N} g(y_i)\,\Delta y$

$y_i = 0 + i\Delta y$

$\Delta y = \frac{1-0}{n} = \frac{1}{n}$

$1 - \lim_{N\to\infty} \sum_{i=y} \frac{i^2}{N_2}\left(\frac{1}{N}\right)$

$1 - \lim_{N\to\infty} \frac{1}{N^3}\left(\frac{N(N+1)(2n+1)}{6}\right.$

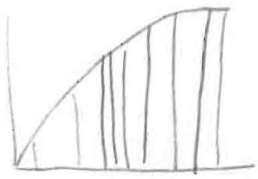
$1 - \frac{1}{3}$

$= \boxed{2/3}$

TEST QUESTION

* let partition ... be 1, 3, B, 2 ... let $c_i = \frac{1}{2}$, ... find Riemann sum

Riemann



* can be any size rectangles

* Can measure rectangles from any point

$\lim_{\|x\|\to 0} \sum_{i=1}^{N} f(c_i)\Delta x_i$

$x_{i-1} \le c_i \le x_i$

$\Delta X_i = X_i - X_{i-1}$

$\|x\|$ Norm

$\|x\|$ largest ΔX_i

$c_i = \frac{i^2}{n^2}$

c_i: $X_i = \frac{i^2}{n^2}$

$n = 5$

$\Delta X_i = \frac{i^2}{n^2} - \frac{(i-1)^2}{N^2} = \frac{i^2 - i^2 + 2 \ldots}{N^2}$

$\lim_{\|x\|\to 0} \sum_{i=1}^{N} \sqrt{\frac{i^2}{N^2}}\left(\frac{2i-1}{N_2}\right)$

$\lim_{n\to\infty} \sum_{i=1}^{N} \left(\frac{2i^2}{N^3} - \frac{i}{N^3}\right)$

$= \frac{2i-1}{N^2}$

$\lim_{N\to\infty} \left(\frac{2}{N^3}\left(\frac{N(N+1)(2N+1)}{6}\right) - \frac{1}{N^3}\frac{(N(N+1))}{2}\right)$

$= \frac{2}{3} - 0$

$= \boxed{2/3}$

$\lim_{\|\Delta\|\to 0} \sum_{i=1}^{N} f(c_i)\Delta X_i = \int_a^b f(x)\,dx$

equivalent to $n\to\infty$

Theorem: Continuity implies integrability

* + differentiability implies integrability

$$\int_{-1}^{3} x^2 dx = \lim_{\|x\| \to 0} \sum_{i=1}^{N} f(c_i)\Delta x_i$$

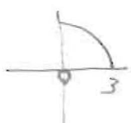
$$\Delta x_i = \Delta x = \frac{3-(-1)}{N} = \frac{4}{N}$$

$$c_i = x_i = -1 + i\Delta x = -1 + \frac{4i}{n}$$

$$\lim_{N \to \infty} \left(-1 + \frac{4i}{N}\right)^2 \frac{4}{N}$$

etc...

$$\int_0^3 \sqrt{9-x^2}\, dx = \frac{1}{4}\pi (3)^2 = \frac{9}{4}\pi$$



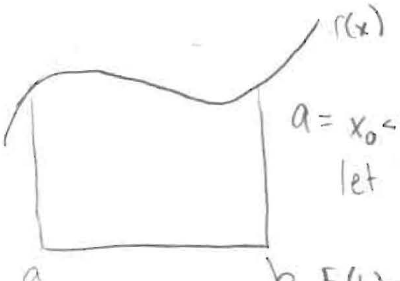
4.4. BIGGEST MOMENT IN CALC.

- Fundamental Theory of calculus

- If $F(x)$ is an antiderivative of $f(x)$, then $\int_a^b f(x) = F(b) - F(a)$

$$\int_0^1 \sqrt{x}\, dx = \frac{2}{3} x^{3/2} \Big]_0^1 = \frac{2}{3} - 0 = \frac{2}{3}$$

- it is area if going L to R in a positive function



$a = x_0 < x_1 < x_2 < \ldots < x_{i-1} < \ldots x_{N-1} < x_N = b$

let $F(x)$ be an antiderivative of $f(x)$ ie $F'(x) = f(x)$

$$F(b) - F(a) = F(x_n) - F(x_{N-1}) + F(x_{N-1}) - F(x_{N-2}) + F(x_{N-2}) \ldots F(x_i) \cdot F$$

$$= \sum_{i=1}^{N} \underbrace{\left(f(x_i) - f(x_{i-1})\right)}$$

$f(x) \rightarrow$

$$F'(c_i) = \frac{F(x_i) - F(x_{i-1})}{x_i - x_{i-1}}$$

Δx

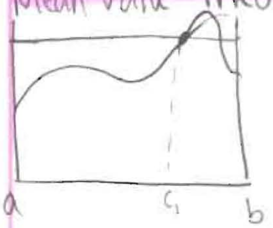
same

$x_{i-1} < c_i < x_i$

$$= f(c_i)\Delta x = \underbrace{F(x_i) - F(x_{i-1})}$$

$$\therefore \sum_{i=1}^{N} f(c_i)\Delta x \qquad \therefore \lim_{\substack{N \to \infty \\ \text{or} \\ \|\Delta\| \to 0}} \sum_{i=1}^{N} f(c_i)\Delta x_i \qquad \therefore \int_a^b f(x) dx = F_b - F_a$$

Mean Value Theorem for Integrals



$$\int_a^b f(x)dx = f(c)(b-a)$$

$$\boxed{f(c) = \frac{\int_a^b f(x)dx}{b-(a)}}$$

$$F'(c) = \frac{F(b)-F(a)}{b-a}$$

average value of a function over a certain interval

* if $||\Delta|| \to 0$, then $n \to \infty$
* if $n \to \infty$, $||\Delta||$ does NOT have to go $\to \infty$

$$\int_a^a f(x)dx = 0$$

$$\int_a^b K f(x)dx = K\int_a^b f(x)dx$$

$$\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

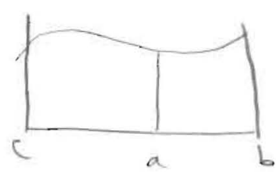
$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$\lim_{||\Delta|| \to 0} \sum_{i=1}^{N} f(c_i)\Delta x_i$ | $-\lim_{||\Delta|| \to 0} \sum_{i=1}^{N} f(c_i)\Delta x_i$

$\Delta x = \frac{b-a}{N}$ | $-\Delta x = \frac{a-b}{N}$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

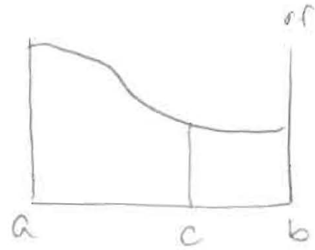
* True regardless of position of a, b, c



$$\int_c^b f(x)dx = \int_c^a f(x)dx + \int_a^b f(x)dx$$

$$-\int_c^a f(x) + \int_c^b f(x)dx = \int_a^b f(x)$$

$$= \int_a^c f(x) + \int_c^b f(x) =$$



$$\int_a^c f(x)dx + \int_c^B f(x)dx = \int_a^b dx$$

find example

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$G(x) = \int_a^x f(t)\,dt = F(x) - F(a)$$
$$G'(x) = F'(x) - 0 \qquad F'(x) = f(x)$$

$$g(x) = \int_0^x t^2\,dt = \frac{t^3}{3}\Big]_0^x = \frac{x^3}{3}$$

AP Nice

$$g(x) = \int_3^x \cos(t^2)\,dt$$
$$g'(x) = \cos(x^2)$$

AP Mean

use chain rule

$$g(x) = \int_3^{x^2} \cos t^2\,dt$$
$$g'(x) = \cos(x^2)^2(2x)$$
$$= 2x\cos x^4$$

Rectilinear Motion

$x(t) = t^3 - 6t^2 + 9t - 2 \qquad 0 \le t \le 5$

$v(t) = x'(t) = 3t^2 - 12t + 9$
$= (t-1)(t-3)$

$v'(t) = a(t) = 6t - 12$
$= 6(t-2)$

t		1		3	
3	+ +		+		+
$t-1$	− −		+ + +		+
$t-3$	− −		− − −		+
v	+		−		+
a	−				

- particle is at rest when v = 0
  $t = 1 \quad t = 3$
- moving right 0−1, 3−5
- when acceleration + v are the same sign, then speed up

$x(0) = -2$
$x(1) = 2$ > 4
$x(3) = 27 - 54 + 27 - 2 = -2$ > 4
$x(5) = 5(25) - 6(25) + 45 - 2 = 18$ > 20

$\boxed{X_T = 28}$

OR

$\int_0^5 |v(t)|\,dt = \int_0^5 |3t^2-12t+9|\,dt$

$\int_0^1 (3t^2-12t+9)\,dt + \int_1^3 -(3t^2-12t+9)\,dt + \int_3^5 (3t^2-12t+9)\,dt$

* To put in calc: store $3t^2-12t+9$
Math 9
fnInt(abs(Y_1), X, 0, 5)

$\int \sin 2x\,dx$ $\quad u = 2x$
$=$ $\quad du = 2\,dx$
$\frac{1}{2}\int 2\sin 2x\,dx$
$= -\frac{1}{2}\cos u + C$ $\quad u = 2x$
$= \boxed{-\frac{1}{2}\cos 2x + C}$ $\quad du = 2\,dx$

$\int \sin 2x\,dx$
$-\int -\sin x\cos x\,dx = -\int -2\cos x(-\sin x\,dx)$
$u = \cos x$
$du = -\sin x\,dx$
$-\int 2u\,du$
$u^2 + C$
$\boxed{-\cos^2 x + C}$

$\int \sin 2x\,dx$
$\frac{1}{2}\int 2\sin x\cos dx = \boxed{\frac{1}{2}\int 2u\,du}$
$u = \sin x$
$du = \cos x\,dx$
$\frac{1}{2}u^2 + C$
$\frac{1}{2}\sin^2 x + C$

* all answers are correct

$\int x(x^2+1)^{1/2}\,dx$
$\int x\sqrt{x^2+1}\,dx$
$u = x^2+1 \quad du = 2x\,dx$
$\frac{1}{2}\int 2x\sqrt{x^2+1}\,dx$
$= \frac{1}{2}\int \sqrt{u}\,du$
$= \frac{1}{3}\left(\frac{u^{3/2}}{3/2}\right) + C = \frac{1}{3}(x^2+1)^{3/2} + C$

$\int (x^2+1)^{1/2}\,dx$
$\int \sqrt{x^2+1}\,dx$

$\int (x^2+2x+1)^{1/2}\,dx$
$\int \sqrt{(x+1)^2}\,dx$
$\int |x+1|\,dx$

$\int x\sin x^2\,dx$
$u = x^2$
$du = 2x\,dx$
$\frac{1}{2}\int 2x\sin x^2\,dx = \frac{1}{2}\int \sin u\,du \quad -\frac{1}{2}\cos 2u + C$

Even n+d rule

$\int_1^3 \sqrt{2x-1}\,dx$

$u = 2x-1$

$du = 2\,dx \quad = \frac{1}{2}\int_1^3 2\sqrt{2x-1}\,dx \qquad 2(1)-1=1$

$= \frac{1}{2}\int_1^5 \sqrt{u}\,du \qquad 2(3)-1=5$

$= (\frac{1}{2})(\frac{2}{3})u^{3/2}\Big]_1^5$

$= \frac{1}{3}(5^{3/2}-1)$

Trapezoid Rule

$\int_a^b f(x)\,dx$

$\frac{1}{2}h$

$\frac{1}{2}\Delta x\,(fx_0 + f(x_1)) + \frac{1}{2}\Delta x\,(f(x_1)+f(x_2)) + \frac{1}{2}\Delta x(fx_2 + fx_3 \ldots$

$\frac{1}{2}\Delta x(fx_{n-1} + fx_n)$

$=$

$\int_a^b f(x)dx \approx \frac{b-a}{2n}(fx_0 + 2fx_1 + 2fx_2 + 2fx_3 + 2fx_{n-1} + fx_n)$

Error $\le \frac{(b-a)^3}{12N^2}\max|f''(x)| \quad a \le x \le b$ $\quad$ *if concave up, overestimation

use closed interval test

If f is even

$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)\,dx$

if f is odd

$\int_{-a}^{a} f(x)\,dx = 0$

Trapezoid cont

$\int_0^1 \sin x^2\,dx \quad$ -4 regular partitions

$\frac{(1-0)}{2(4)}\left(\sin 0^2 + 2\sin\frac{1}{16} + 2\sin\frac{1}{4} + 2\sin\frac{9}{16} + \sin 1\right)$

$y' = 2x\cos(x^2)$

$y'' = 2\cos x^2 + -4x^2\sin x^2$

$E \le \frac{(1-0)^3}{12(16)}(6) = \frac{1}{32}$

rough estimate

$\int_1^2 \frac{1}{x}\,dx \quad = 4$ partitions

$\frac{(2-1)}{2(4)}\left(f(1) + 2f(5/4) + 2f(\frac{3}{2}) + 2f(\frac{7}{4}) + f(2)\right)$

$\frac{1}{8}\left(1 + 2(\frac{4}{5}) + 2(\frac{2}{3}) + 2(\frac{4}{7}) + \frac{1}{2}\right)$

$\frac{1}{8}\left(1 + \frac{8}{5} + \frac{4}{3} + \frac{8}{7} + \frac{1}{2}\right) = \frac{1171}{1681}$

Cont →

Error $y = \frac{1}{x} = x^{-1}$

$y' = -1x^{-2}$

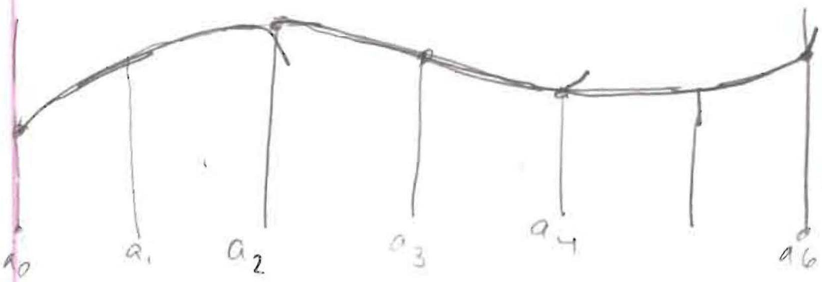
$y'' = 2x^{-3}$

$y''' = -6x^{-4}$

$E \leq \frac{(2-1)^3}{12(16)}(2) = \frac{1}{96}$

$\int_a^b (Ax^2 + Bx + C)\,dx = \frac{b-a}{6}\left(P(a) + 4P\left(\frac{a+b}{2}\right) + P(b)\right)$ Simpson's Rule

* must partition even number tims



$\int_a^b f(x)\,dx \approx$

$\frac{2\Delta x}{6} = \frac{2\frac{(b-a)}{N}}{6} = \frac{(b-a)}{3n}$

$\frac{2\Delta x}{6}\left(f(x) + 4f(x_1) + 2f(x_2) + 4f(x_3)\right.$

$+ 4f(x_5) + f x_6 \ldots$

$\frac{b-a}{3N}\left(f(x_0) + 4f(x_1) + 2(fx_2) + 4f(x\right.$

$4f(x_{N-1}) + f(x_N)$

* must end with coefficients 4 + 1

* Coefficient pattern 1, 4, 2, 4, 2, 4, 2, 4, 1

Error Analysis

$E \leq \frac{(b-a)^5}{180N^4} \max |f^4(x)| \quad a \leq b$

$\left.\begin{array}{l}\int_a^b f'(x)\,dx = f(b) - f(a) \\ \int_a^b f(x)\,dx = F(b) - F(a)\end{array}\right\}$ same

AP: $f'(x) = \cos x^2$

$f(1) = 3$

$f(2) = ?$

$\int_1^2 \cos x^2 = f$

9 mc

- understanding trap. rule + Riemann
- def/indef int
- fundamental
- Substitution

Data Chart

- dont need to use all #s
- left/right/mid Rieman sums
- trap + simp. rule

Page of Integrals

- use subs

Know how solve w/ def.

Error

- little partial credit
- * Answer is a nice, simple fraction

Chapter 5

$f(x) = \int_1^x \frac{1}{t}\,dt$

$f(1) = 0$ (b/c from 1 to 1 is zero)

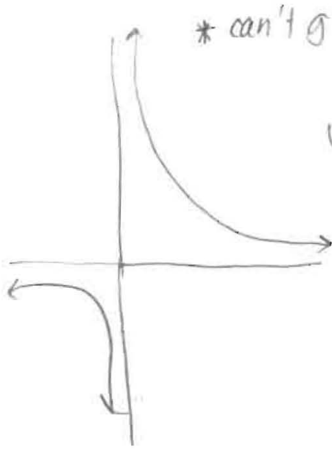
$f'(x) = \frac{1}{x}$

D_f $(0, \infty)$

R $(-\infty, \infty)$

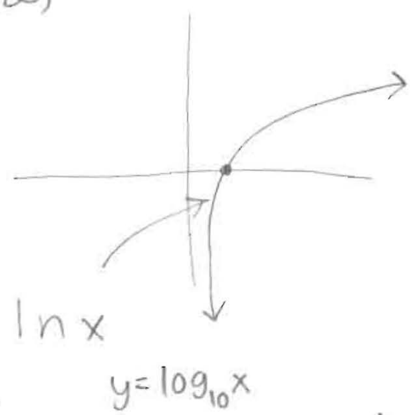
$f''(x) = \frac{-1}{x^2}$

* can't go through zero



$\int_1^{1/2} \frac{1}{t}\,dt = -\text{Area}$

$\int_1^{.1} \frac{1}{t}\,dt = -\text{Area}$



$\ln x$

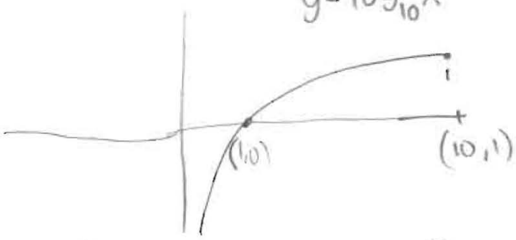
$g(x) = \int_0^x \sin t\,dt$

$g(x) = -\cos t \Big]_0^x$

$g(x) = -\cos x - (-1)$

$1 - \cos x$

$y = \log_{10} x$

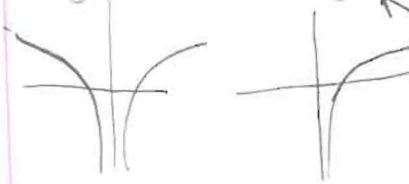


* log graphs are nearly flat

look up when to put abs. value

$\log x^2 = 2\log x$ LIE

really should be $2\log|x|$



rule should be $\log(MN) = \log|M| + \log|N|$

oof: $\ln(ab) = \ln a + \ln b \quad a>0 \quad b>0$

$f(x) = \ln(ax) \qquad g(x) = \ln a + \ln x$

$f'(x)\ \frac{1}{ax}(a) \qquad g'(x) = 0 + \frac{1}{x}$

$\frac{1}{x} \qquad \frac{1}{x}$

$\therefore$ functions differ by a constant

$\ln(ax) = \ln a + \ln x + C$

$\ln(a \cdot 1) = \ln a + \ln 1 + C$

$\therefore C = 0$

Proof: $\log_a m = x \qquad \log_a N = y$

$a^x = m \qquad a^y = N$

$NN = a^x a^y$

$MN = a^{x+1}$

$\log_a(mn) = x+y$

$\log_a(mn) + \log_a m + \log_a N$

Rule: $\ln\left(\frac{a}{b}\right) = \ln a - \ln b \quad a>b,\ b>0$