4.1

$f'(x) = F(x)$ $\qquad$ $f(x) = g(x) + c$

$g'(x) = F(x)$ $\qquad$ $h(x) = f(x) - g(x)$ $\quad$ assume $h(x)$ is not constant

$h(a) \neq h(b)$

$\frac{h(b) - h(a)}{b-a} \neq 0$

MVT $\quad h'(c) = \frac{h(b) - h(a)}{b-a}$ $\qquad$ $h'(x) = F(x) - F(x) = 0$

$\therefore$ assumption is wrong

if two functions have same derivative, only differ by a constant

Antiderivatives/Integrals

$\int 0\,dx$ = Constant

$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1$

$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

$\int kf(x)\,dx = k \int f(x)\,dx$

$\int \sin x = -\cos x + c$

$\int \cos x\,dx = \sin x + c$

$\int \sec^2 x\,dx = \tan x + c$

$\int \csc^2 x\,dx = -\cot x + c$ $\qquad$ $\int k\,dx = kx + c$

$\int \sec x \tan x\,dx = \sec x + c$

$\int \csc x \cot x\,dx = -\csc x + c$

$$\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{x^{-2}}{-2} + C = \frac{-1}{2x^2} + C$$

$$\int \tan^2 x\,dy = \int (\sec^2 x - 1)\,dx = \tan x - x + C$$

* Dont forget the C!

$a = -32\ ft/s^2$

$V = \int -32\,dt = -32t + C_1$

$V(0) = C_1 = V_0$

$V = -32t + V_0$

$h = \int (-32t + V_0)\,dt$

$h(t) = -\frac{32t^2}{2} + V_0 t + C_2$

$h(0) = C_2 = h_0$

$h(t) = -16t^2 + V_0 t + h_0$

$$\sum_{i=1}^{N} a_i = a_1 + a_2 + a_3 + \ldots + a_{n-1} + a_n$$

summation

$$\sum_{i=3}^{5} (i^2) = 9 + 16 + 25 = 50$$

$$\sum_{i=1}^{100} i =$$

$$\sum_{i=1}^{N} k = Nk$$

$$\sum_{i=1}^{N} i = 1 + 2 + 3 + 4 \ldots + N-1 + N$$

$S = 1 + 2 + 3 + 4 + \ldots N-2 + N-1 + N$

$S = N + 1 + N-1 + N-2 \quad 3 \quad 2 \quad 1$

$2S = (N+1) + (N+1) + (N+1) + (N+1)$

$= \boxed{S = \frac{N(N+1)}{2}}$

Prove by induction:

- if P_1 is true
- if P_k is true implies P_{k+1} is true
- then P_N is true for all N

$$\sum_{i=1}^{N} i^2 = \frac{N(N+1)(2N+1)}{6}$$

$$\sum_{i=1}^{1} i^2 = 1^2 = 1 \quad \frac{1((1+1)(2(1)+1)}{6} = 1$$

-Assume P_k is true

$$\sum_{i=1}^{k+1} \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

CONTINUED →

$$\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2$$

$$\frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)^2}{6}$$

$$= \frac{(k+1)(2k^2+k+6k+6)}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

the formula

$$\sum_{i=1}^{N} i^3 = \frac{N^2(N+1)^2}{4}$$

$$\sum_{i=1}^{N} (a_i + B_i) = \sum_{i=1}^{N} a_i + \sum_{i=1}^{N} B_i$$

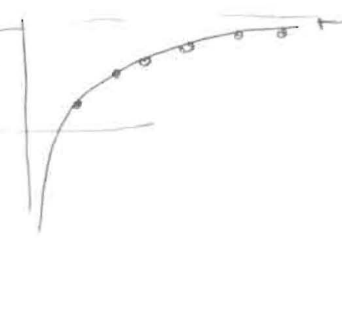
* No product rule

$$\sum_{i=1}^{N} k a_i = k \sum_{i=1}^{N} a$$

$$\lim_{N\to\infty} \sum_{i=1}^{N} a_i$$

if $f(x) = a_N$ for all $x = n$ and $\lim_{x\to\infty} f(x) = L$, then $\lim_{x\to\infty} a_n = L$

- if limit on series, then function has limit

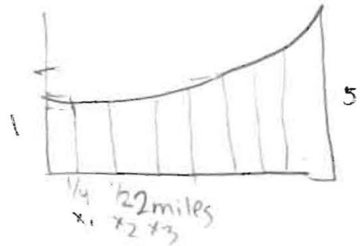


will approach horizontal limit

converse: not true

$f(x) = \sin \pi x$

$a_N = \sin \pi (0)$ ← will always be zero



$y = x^2 + 1 \qquad X_0 = 0 \qquad x_1 = 0 + \frac{1}{4} \quad x_2 = 0 + 2(\frac{1}{4})$

$x_i = \frac{1}{4} i$

$x_{i-1} = \frac{1}{4}(i-1)$

$f(\frac{1}{4})(\frac{1}{4}) + f(\frac{1}{2})(\frac{1}{4})$

OR

$\Delta x = \frac{2-0}{N}$

$X_0 = 0$

$X_1 = 0 + \frac{2}{N}$

$X_2 = 0 + 2(\frac{2}{N})$

$X_i = (\frac{2}{N}) i$

$\lim_{N\to\infty} \sum_{i=1}^{N}$

$\sum_{i=1}^{N} f(\frac{2}{5} i)(\frac{2}{N})$

$\lim_{N\to\infty} \sum_{i=1}^{N} \left(\frac{4i^2}{N^2} + 1\right)\left(\frac{2}{N}\right)$

- measured from right

$\lim_{N\to\infty} \sum_{i=1}^{n} \left(\frac{8i^2}{N^3} + \frac{2}{N}\right)$

$\lim_{N\to\infty} \frac{8}{N^3} \frac{(N(N+1)(2N+1))}{6} + \frac{2N}{N}$

$= \frac{8}{3} + 2$

$= \frac{14}{3} = 4\frac{2}{3}$

$\lim \sum_{i=1}^{N} (X_{i-1}) (\Delta X$

$\lim_{N\to\infty} \sum_{i=1}^{n} f\left(\frac{2}{N}(i-1)\right)\left(\frac{2}{N}\right)$

-measured from left

$\lim_{N\to\infty} \sum_{i=1}^{N} \left(\frac{4}{N^2}(i^2 - 2i + 1) + 1\right)\left(\frac{2}{N}\right)$

$\lim_{N\to\infty} \sum_{i=1}^{N} \left(\frac{8i^2}{N^3} - \frac{16i}{N_3} + \frac{8}{N^3} + \frac{2}{N}\right.$

$\lim_{N\to\infty} \left(\frac{8}{N_3}\left(\frac{N(N+1)(2N+1)}{6}\right) - \frac{16}{N^3}\left(\frac{N(N+1)}{2}\right) + \frac{8}{N^3} N\right.$

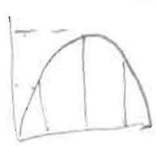
$= 8/3 - 0 + 0 + 2$

$y = \sin x \; [0, \pi]$

$A_L = 0(\pi/4) + \frac{\sqrt{2}}{2}(\frac{\pi}{4}) + 1(\frac{\pi}{4}) + \frac{\sqrt{2}}{2}(\frac{\pi}{4})$

$= \frac{\pi}{4}(\sqrt{2} + 1)$

- right + left balance, but still doesn't mean right area



Midpoint

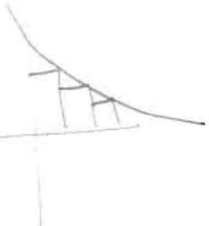
$$\frac{\pi}{4}\left(\sin\frac{\pi}{8} + \sin\frac{3\pi}{8} + \sin\frac{5\pi}{8} + \sin\frac{7\pi}{8}\right)$$

$$= 2.05$$

Vocab:

inscribe - inside boxes

- underaproximation



or

circumscribe

- over approximation