19 Oct 2009

35)

$$V(x,y) = \frac{1}{3}\pi x^2 (r+y)$$

$$x^2 + y^2 = r^2$$

$$V(y) = \frac{1}{3}\pi (r^2 - y^2)(r+y) \qquad 0 \le y \le r$$

$$V'(y) = \frac{\pi}{3}\left(-2y(r+y) + 1(r^2-y^2)\right)$$

$$\frac{\pi}{3}\left(-2yr - 2y^2 + r^2 - y^2\right)$$

$$\frac{\pi}{3}\left(r^2 - 2yr - 3y^2\right)$$

$$\frac{\pi}{3}(r+y)(r-3y)$$

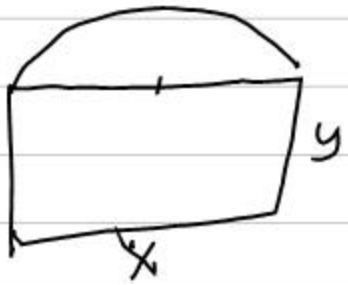
$$y \neq -r \qquad y = \frac{1}{3}r$$

$$V(0) = \frac{\pi}{3}r^3$$

$$V(r) = 0$$

$$V\left(\frac{1}{3}r\right) = \frac{1}{3}\pi\left(\frac{8}{9}r^2\right)\left(\frac{4}{3}r\right) = \frac{32\pi r^3}{81}$$

23)



$16 = x(1+\pi/2)$

$\frac{32}{2+\pi}$

$$16 = 2y + x + \frac{\pi x}{2} \Rightarrow y = 8 - \frac{x}{2} - \frac{\pi x}{4}$$

$$A(x,y) = xy + \frac{\pi}{2}\left(\frac{x}{2}\right)^2$$

$$A(x) = 8x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8} \qquad 0 \le x < \frac{32}{2+\pi}$$

$$A'(x) = 8 - x - \frac{\pi}{2}x + \frac{\pi}{4}x$$

$$A''(x) = -1 - \pi/2 + \pi/4 = -1 - \pi/4 < 0$$

∴ Any critical x will yield a MAX by 2nd deriv. test

$A'(x) = 0$ if $\qquad 8 = x\left(1 + \frac{\pi}{2} - \frac{\pi}{4}\right)$

$$x = \frac{32}{4 + 2\pi - \pi} = \boxed{\frac{32}{4+\pi}}$$