6 Oct 2009

36) $f(x) = \dfrac{x+3}{x^2}$ $\qquad f'(x) = \dfrac{1(x^2) - 2x(x+3)}{x^4}$

$$\frac{x^2 - 2x^2 - 6x}{x^4}$$

$$\frac{-x^2 - 6x}{x^4}$$

$$\frac{-x(x+6)}{x^4}$$

$$\frac{-(x+6)}{x^3}$$

x int $(-3, 0)$

y int None

VA $x = 0$

HA $y = 0$

inc ~~$(-\infty, -6)$~~ $(-6, 0)$

dec ~~$(-6, 0) \cup (0, \infty)$~~ $(-\infty, -6) \cup (0, \infty)$

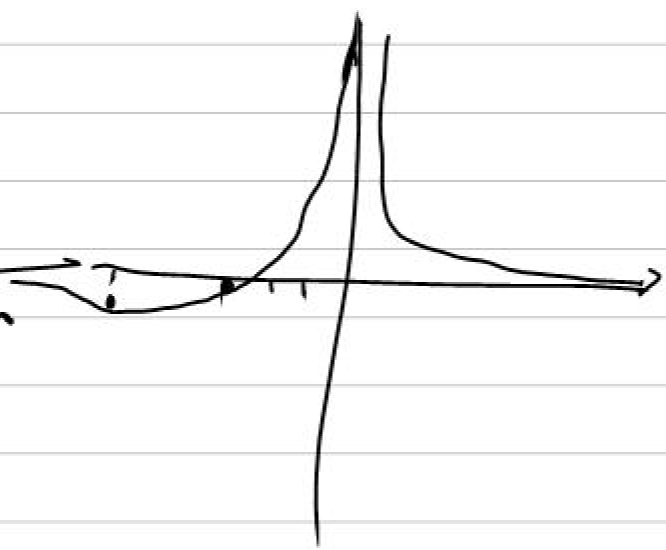
rel max ~~$(-6, -1/12)$~~ None

rel min ~~None~~ $(-6, -1/12)$

critical pt $(-6, -1/12)$

range $[-1/12, \infty)$

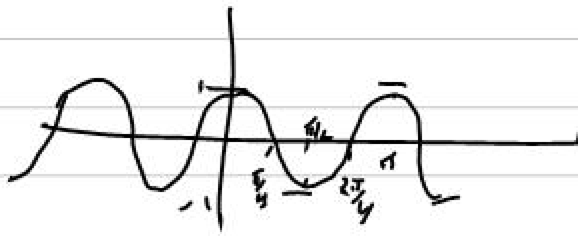
x		-6		0	
-1	$- - - -$		$- - - -$		$- -$
$x+6$	$- - - -$	0	$+ + +$		$+ + + + +$
x^3	$- - -$		$- - -$	0	$+ + + +$
y'	$- - -$	0	$+ + +$	ND	$- - - -$
y	dec		inc		dec



f has a rel min at $x = -6$ because $f'(x) < 0$ for $x < -6$ and $f'(x) > 0$ for $x > -6$

54) $f(x) = \cos^2 t - \sin^2 t$

$g(x) = 1 - 2\sin^2 t$ $\Big\}$ $\cos 2t$



$f'(x) = 2\cos t(-\sin t) - 2\sin t\cos t$

$- 4\sin t\cos t$

t		-2π		$-3\pi/2$		$-\pi$		$-\pi/2$		0		$\pi/2$		π		$3\pi/2$		2π
-4	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−
$\sin t$		0	+	+	+	0	−	−	−	0	+	+	+	0	−	−	−	0
$\cos t$	+	+	+	0	−	−	−	0	+	+	+	0	−	−	−	0	+	
		0		0		− 0		+ 0		− 0		+ 0		− 0		+ 0		

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f is concave upward if $f''(x) > 0$ (i.e f' is inc)

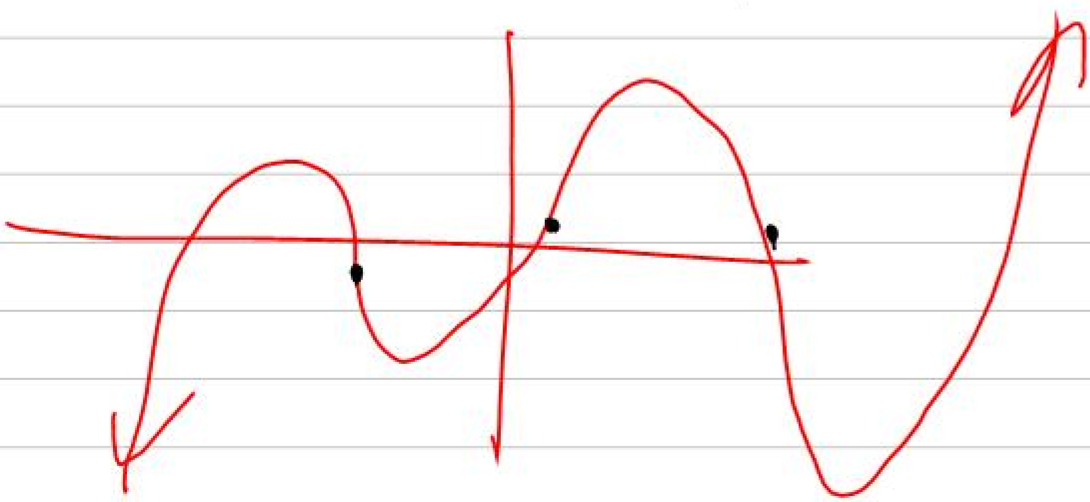
f is concave downward if $f''(x) < 0$ (i.e. f' is dec)

if f changes concavity at $(c, f(c))$

then $(c, f(c))$ is a point of inflection.

Concave up

Concave down



2nd derivative test

~~if $f'(c)=0$~~ if $(c, f(c))$ is a critical pt

$(c, f(c))$ is a rel MAX if $f''(c) < 0$

$(c, f(c))$ is a rel MIN if $f''(c) > 0$

if ~~not~~ $f''(c) = 0$ then the test fails

$(c, f(c))$ is critical $\Rightarrow f'(c) = 0$

if $f''(c) > 0$ then $\dfrac{f'(x) - f'(c)}{x - c} > 0$ for x close to c

$f'(x) - f'(c) > 0$

if $x > c$ $\quad f'(x) > f'(c) \Rightarrow f'(x) > 0$

if $x < c$ $\quad f'(x) - f'(c) < 0 \Rightarrow f'(x) < 0$

$f'(x) < f'(c)$